Indian Statistical Institute, Bangalore Centre
B.Math. (II Year) : 2009-2010
Semester II : Semestral Examination
Optimization

30.04.2010 Time: 3 hours. Maximum Marks : 100

Note: The paper carries 105 marks. Any score above 100 will be taken as 100.

1. [8 + 15 marks] (i) Let $F_1 \subset I\!R^k, F_2 \subset I\!R^p$ be convex sets. Suppose $H : I\!R^k \to I\!R^p$ is a linear transformation such that H establishes a $1-1$ correspondence between F_1 and F_2. Show that there is a $1-1$ correspondence between the extreme points of F_1 and F_2.

(ii) Let $m \leq n$. Consider the LP: *Minimize* $c^T x$ subject to $Ax \leq b, x \geq 0$, where A is an $(m \times n)$ real matrix of rank m, $b \in I\!R^m, c \in I\!R^n$. Let F be the feasible set for the LP. Let $\tilde{F}$ denote the feasible set for the equivalent LPS. Show that there is a $1-1$ correspondence between the extreme points of these two problems.

2. [15 marks] Consider the LP: *Minimize* $(-2x_1 - x_2)$ subject to $x_1 + x_2 \leq 3, \quad 2x_1 + x_2 \leq 5, \quad x_1 \geq 0, \quad x_2 \geq 0$. Convert this into standard form, and solve it using the simplex method.

3. [14 + 3 marks] (i) *Maximize*

$$f(x) = \exp\{-\frac{1}{2}(x-\mu)^T Q(x-\mu)\}, \quad x \in I\!R^n,$$

where $\mu \in I\!R^n$ and Q is an $(n \times n)$ real symmetric strictly positive definite matrix. Give proper justification for each step.

(ii) In (i) above, what is the minimum value of $f(\cdot)$? Is it attained?

4. [10 + 10 marks] (i) Using the method of Lagrange multipliers, and justifying its use, *maximize* $(x^2 - y^2)$ subject to the equality constraint $(x^2 + y^2) = 1$. Is the global maximum point unique?

(ii) Consider the constrained optimization problem: *Maximize* $(x^2 - y^2)$ subject to $(x + y) = 1$. Will it be fruitful to apply the method of Lagrange multipliers? Why?

5. [12 + 18 marks] Consider the constrained optimization problem: *Maximize* $[(x+1)^2 + (y+1)^2]$ subject to $x^2 + y^2 \leq 2, \quad y \leq 1$.

(i) Justify why the Kuhn-Tucker method will work in this case.

(ii) Solve the problem using the Kuhn-Tucker method.